IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Random walk on self-avoiding walk in external bias: diffusion, drift and trapping

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1985 J. Phys. A: Math. Gen. 18 L761

(http://iopscience.iop.org/0305-4470/18/13/006)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 31/05/2010 at 08:56

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. 18 (1985) L761-L766. Printed in Great Britain

LETTER TO THE EDITOR

Random walk on self-avoiding walk in external bias: diffusion, drift and trapping

Debashish Chowdhury†

Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, D-5000 Köln 41, West Germany

Received 17 June 1985

Abstract. The effect of external bias on random walks (RWs) on self-avoiding walks (on square lattices) in the presence of 'bridges' is studied by Monte Carlo simulation. The qualitative as well as quantitative feaures of the latter problem are compared and contrasted with those of RWs on percolation clusters in the presence of external bias.

Recently the phenomenon of random walks (RWs) on statistical fractals, e.g., on the percolation clusters (PCs) (see, e.g., Stauffer 1985a for a review) and on self-avoiding walks (SAWs) (Helman *et al* 1984, Ball and Cates 1984, Chowdhury 1985, Yang *et al* 1985, Chowdhury and Chakrabarti 1985, from now on the latter work will be referred to as I), have been studied intensively. The non-triviality of the latter problem arises from the hoppings across the so-called bridges (see I for the details). What happens when an external biasing field is imposed? Suppose, bias $B=0$ and $B=1$ (in dimensionless units) correspond, respectively, to the completely unbiased and completely biased situations. The mean square end-to-end distance of a RW after 'time' t will be denoted by $\langle R_t^2\rangle$. In the case of RWs on PCs above percolation threshold the following features have emerged.

(i) For a given non-zero finite (not too large) bias B, a crossover from diffusive to drift-like motion takes place at a value t_{cr} of time (Pandey 1984).

(ii) t_{cr} is a decreasing function of the biasing field B (Pandey 1984).

(iii) For a given t, $\langle R_t^2\rangle$ is a non-monotonic function of B, a maximum occurs at an intermediate value of B (Barma and Dhar 1983, Pandey 1984).

(iv) For a given B, the component of the end-to-end distance parallel to the bias increases linearly with time (Seifert 1984) for sufficiently large t.

(v) For a given B, the component of the end-to-end distance perpendicular to the bias increases as $t^{1/2}$ with time (Seifert 1984) for sufficiently large t.

(vi) At $B=1$, $\langle R_t^2\rangle$ is independent of t, the latter effect is a consequence of the creation of traps by the field (Böttger and Bryksin 1982, Barma and Dhar 1983, Pandey 1984, Seifert and Suessenbach 1984, Stauffer 1985b). The traps arise from the fact that moving out of the 'dead-end branches' and the 'cages' in the PC may require motion against the field.

† Alexander von Humboldt Postdoctoral Fellow.

0305-4470/85/130761+06\$02.25 © 1985 The Institute of Physics

However, what remains controversial is the possibility of drift even in the presence of finite, but large, non-zero values of B (Dhar 1984, Stauffer 1985b, Ohtsuki 1982, White and Barma 1984, Gefen and Goldhirsch 1985). Our main aim in this letter is to study the effect of an external biasing field on the mean square end-to-end distance, $\langle R_t^2 \rangle$, of a t-step RW on a longer SAW, because 'cages', similar to those leading to trapping on PC, occur also on SAWs. However, the most remarkable difference between the two problems is the absence of 'dead-end branches' on a SAW. Fortunately, this difference can be indirectly utilised to compare and contrast the roles played by the 'cages' and the 'dead-end branches' in the case of RWs on PCs.

In I we assumed the random walker to be a 'myopic ant' (Mitescu and Roussenq 1983); the time step in such a walk does not increase if the next host site chosen by the ant is inaccessible. In the present work we shall assume the walker to be a 'blind ant'; the time step is increased even if the attempt for a step is unsuccessful. In case of RW on PC, both the 'myopic ant' and the 'blind ant' lead to the same critical behaviour (see, e.g., Seifert and Suessenbach 1984). Our present choice of the ant is motivated by our aim of studying the trapping of the ant (the distance traversed by the ant must not increase with the increase of time, at least over a sufficiently large time interval, if it is trapped). A myopic ant is not suitable for the latter purpose because, when trapped, both the distance and time do not increase further. We shall also show in this letter that in the absence of an external biasing field the critical behaviour of the RW of a blind ant on SAW is identical with that of a myopic ant.

We have generated SAWs of lengths $N = 55$ (in a CDC Cyber 72 scalar computer), $N = 75$ and $N = 90$ (in a CDC Cyber 176 scalar computer) on square lattices by Monte Carlo simulation as described in I. About 16 seconds of CPU time was required to generate a SAW of $N = 75$ and about 80 seconds of CPU time for a SAW of $N = 90$ in the CDC 176 computer by our algorithm. We verified that the mean square end-to-end distance exponent for these SAWs is 0.75. An ant is placed on an arbitrary site on a SAW and the magnitude and direction of the field B $(0 \leqslant B \leqslant 1)$ are specified as in Pandey (1984). Suppose B is applied along the south. Now a random fraction between zero and unity is called. The next position of the ant is chosen in the following way.

(i) If the random fraction is less than B the nearest neighbour on the south of the current site is investigated and the time is increased by unity. If the site so chosen lies on the SAW the ant moves there, otherwise it stays at its old position and a new random fraction is called.

(ii) If the random fraction is larger than B, each of the four directions are equally probable and one of these is chosen randomly for investigation by calling another random number, then the time is increased by unity and the ant moves as above. The end-to-end distance after a given interval of time t is measured by following the same procedure as described in I. A large number of configurations (more than 30 000) for each t and B were generated by varying the initial position of the ant on a given SAW and by generating RWs on different SAWs. Less than one microsecond of CPU time was required for each step of the RW. Notice that most of the computer time goes in producing the SAWs. The square of the end-to-end distance of a t-step RW for a given B was averaged over all these configurations to get $\langle R_t^2 \rangle$. The procedure was repeated for various values of t and B. Let us denote the components of the end-to-end distance of a RW parallel and perpendicular to the biasing field by $R_t^{\parallel}$ and $R_t^{\perp}$ respectively. We have also computed the averages $\langle |R_t^{\parallel}| \rangle$ and $\langle |R_t^{\perp}| \rangle$ for the same configurations generated for the computation of the mean square average of the end-to-end distance. The following features emerge.

(i) In the absence of biasing field, the mean square end-to-end distance of t-step RWs on SAWs (the SAW is much longer than the RW) is given by

$$\langle R_t^2 \rangle \propto t^k$$

where $k \simeq 0.72$, in agreement with our earlier observation (see I) using myopic ant. Thus, in the absence of biasing field, the critical behaviour of RWs on SAWs is identical for both myopic as well as blind ants. The latter feature is qualitatively similar to that observed in the case of RWs on PCs (Mitescu and Roussenq 1983). Moreover, the numerical value $k \simeq 0.72$ is smaller than the corresponding value ($k \simeq 0.75$) computed by Yang *et al* (1985) from exact enumeration of RWs on SAWs generated by Monte Carlo simulation. If Yang *et al*'s observation were correct it would mean that the hoppings across the bridges have no effect on the critical behaviour of RWs on SAWs. But our observation here disagrees with Yang *et al*'s claim, thereby providing further support to our earlier argument (Chowdhury 1985 and I) that the hopping across the bridges has a non-trivial effect on the critical behaviour.

(ii) The slope of each of the $\langle R_t^2 \rangle$ against t curves (see figures 1 and 2) starts decreasing beyond a certain value t_{max} and finally vanishes at large t. The flattening of the curves in figures 1(a) and 1(b) for large t is a consequence of the fact that as t increases all the sites on the SAW are likely to be visited. Our subsequent discussion will be confined to times $t \ll t_{max}$. In other words, we shall assume that the end-to-end distance of the RW is much smaller than that of the SAWs.

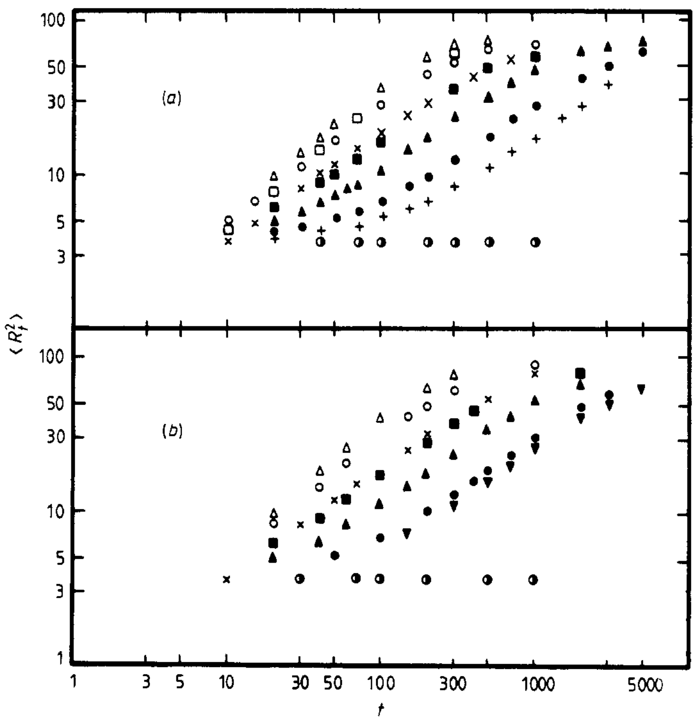

Figure 1. Log-log plots of mean square end-to-end distances $\langle R_t^2 \rangle$ of RWs against t on SAWs of length (a) $N = 55$, and (b) $N = 75$ for various values of the biasing field B (×, ○; □, 0.25; △, 0.50; ○, 0.75; ■, 0.90; ▲, 0.95; ●, 0.98; +, 0.99; ◑, 1.00). The symbol ▼ in (b) corresponds to RWs on SAWs of length $N = 90$ for $B = 0.98$.

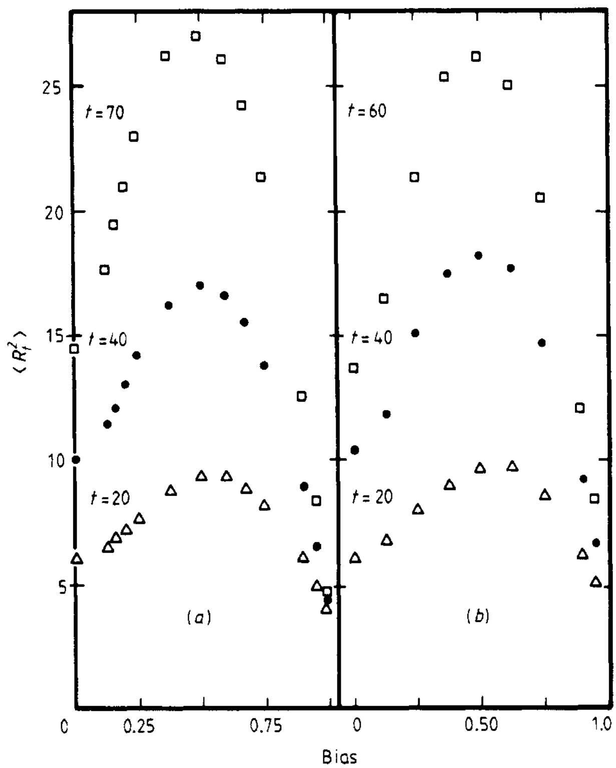

Figure 2. Mean square end-to-end distances $\langle R_t^2 \rangle$ of RWs plotted against the biasing field B on SAWs of length (*a*) $N=55$, and (*b*) $N=75$.

(iii) For a given non-zero finite field B, a crossover to a new critical behaviour takes place at $t=t_{cr}$ (see figures 1(*a*) and 1(*b*)). Notice that the curves for various values of the field are parallel to each other at long times with the slopes approximately 0.67. The latter value of the exponent indicates that, possibly, anomalous diffusion (Gefen *et al* 1983) dominates on long time scale. However, because of the limited observation time ($t<5000$) in our simulation we cannot completely rule out the possibility of truly drift-like (or some other) critical behaviour after sufficiently long time on extremely long SAWs. Although the upward turn of the curves beyond t_{cr} is common for both RW on PC and RW on SAWs there are quantitative differences. Our curves for RWs on SAWs clearly indicate a new power law behaviour for longer times ($t>t_{cr}$); the corresponding curves for RW on PC, at least for large bias (>0.98), appear more complicated (Pandey 1984, Seifert and Suessenbach 1984, Stauffer 1985b). Dhar (1984) argued that in the presence of strong fields the root-mean-square displacement varies as t^{α}, where α ($\alpha<1$) is a non-universal field-dependent exponent. On the other hand, in the case of RW on SAWs we find roughly the same power law behaviour with field-independent exponent for all finite fields.

(iv) The time t_{cr} at which the crossover takes place is, however, clearly a decreasing function of the field B. This feature is also similar to that observed in case of RW on PC (Pandey 1984, Barma and Dhar 1983).

(v) In an infinitely large field ($B=1$) $\langle R_t^2 \rangle$ is independent of t (for all $10<t<5000$) thereby signalling trapping of the ant in the cages. However, no trapping of ants on

SAWS is possible for any finite values of the field. On the other hand, on PC there are dead-end branches, in addition to the cages. If one is allowed to push the analogy between diffusion on SAWS and that on PC too far, one would expect that only the dead-end branches in the PC can give rise to trapping for finite B on the PC, if at all; the 'cages' being insufficient for trapping at finite B. This conjecture is in agreement with the earlier observations on RW on PC (Pandey 1984, Barma and Dhar 1983). Besides, most of the recent studies of biased RW on PC (Dhar 1984, White and Barma 1984, Gefen and Goldhirsch 1985) focus attention on the role of dead-end branches in trapping.

(vi) When observed for a fixed value of t, $\langle R_t^2 \rangle$ is a non-monotonic function of the bias B, as shown in figures 2(a) and 2(b). A small bias pushes the ant forward thereby increasing its end-to-end distance for a given t. However beyond a certain value of B (approximately 0.5 for the square lattice) the ant finds it increasingly difficult to come out of the cages and consequently the end-to-end distance for given t decreases. This behaviour is also very similar to that observed in the case of RW on PC (Pandey 1984, Barma and Dhar 1983). Unfortunately, it has not been possible to decide conclusively on the nature of the asymptotic dependence of the rise (and the subsequent fall) of $\langle R_t^2 \rangle$ with B.

(vii) The t-dependence of $\langle |R_t^{\parallel}| \rangle$ is qualitatively similar to that of $\langle R_t^2 \rangle^{1/2}$. Analogous features have also been observed in the case of RWS on SAWS (Seifert 1984).

(viii) The t-dependence of $\langle |R_t^{\perp}| \rangle$ is, however, more complicated. The effective exponent seems to be approximately unity for sufficiently large, but finite, bias. This, however, is in sharp contrast with the corresponding result for RW on PC (Seifert 1984). Besides, it is hard to justify the value of the exponent physically. We cannot rule out the possibility of a smaller value of the exponent in the large t limit.

Finally, we have reported here some universal features of RWS on SAWS in two dimensions using Monte Carlo simulation. The SAWS are rather short; the longest SAWS simulated are of length $N = 90$. However, simulation of longer RWS on much longer SAWS using a more efficient algorithm will be reported in a future publication.

I thank D Stauffer, B K Chakrabarti and J Kertész for many useful discussions, suggestions and comments. I am indebted to D Stauffer for a critical reading of the manuscript.

References

Ball R C and Cates M E 1984 *J. Phys. A: Math Gen.* **17** L531
Barma M and Dhar D 1983 *J. Phys. A: Math Gen.* **16** 1451
Böttger H and Bryksin V V 1982 *Phys. Stat. Solidi* **b113** 9
Chowdhury D 1985 *J. Phys. A: Math Gen.* **18**
Chowdhury D and Chakrabarti B K 1985 *J. Phys. A: Math Gen.* **18** L377
Dhar D 1984 *J. Phys. A: Math. Gen.* 17 L257
Gefen Y, Aharony A and Alexander S 1983 *Phys. Rev. Lett.* **50** 77
Gefen Y and Goldhirsch I 1985 *Preprint*
Helman J S, Coniglio A and Tsallis C 1984 *Phys. Rev. Lett.* **53** 1195
Mitescu C D and Roussenq J 1983 in *Percolation structures and processes* ed G Deutscher, R Zallen and J Adler p 81
Ohtsuki T 1982 *J. Phys. Soc. Japan* **51** 1493
Pandey R B 1984 *Phys. Rev.* B **30** 489
Seifert E 1984 *Staatsexamensarbeit* Universitat Köln

Seifert E and Suessenbach M 1984 *J. Phys. A: Math. Gen.* **17** L703
Stauffer D 1985a *Introduction to Percolation Theory* (London: Taylor and Francis)
—— 1985b *J. Phys. A: Math. Gen.* **18** 1827
White S R and Barma M 1984 *J. Phys. A: Math. Gen.* **17** 2995
Yang Y S, Liu Y and Lam P M 1985 *Z. Phys.* B in press